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# Nonconservation of Angular Momentum of a Weight on a String

AN oversight in simple mechanics has occasionally occurred in the analysis of the problem of having a weight on the end of a string which is winding up on a steady finger. Several authors of meteorological publications have used this example to illustrate the principal of the conservation of angular momentum, concluding that the speed of the weight increases as the string winds up. This error appeared in the April 1960 issue of *Scientific American*,[1] and was later corrected.[2] It is included in a recent book by Battan[3] (and was not mentioned in the reviews.[4,5]) It remains in the third edition of a text by Byers.[6]

A second look at the problem shows that the angular momentum of the mass–string system is not conserved at all. Angular momentum is defined about a fixed point of reference and the conservation of angular momentum has meaning only with respect to that point. Using the center of the finger as a reference, one finds a torque due to string tension opposing the rotation. Clearly angular momentum cannot be conserved. The common observation of an increase in the "speed" of rotation is the detection of the increase in angular velocity, which increases because the radius shortens and not because of any change in the speed of the weight.

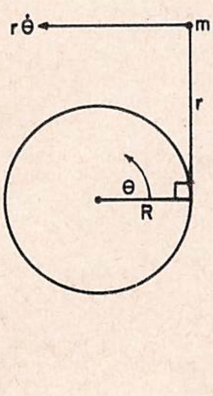


FIG. 1. Coordinates which give easy solution to the angular momentum and trajectory of the mass on a string which is winding up on a finger.

The correct interpretation is easily had by applying the conservation of energy: No work is done on the weight because the finger is stationary. Therefore, the kinetic energy of the weight is constant and so is its speed. If the speed of the weight were to increase, we would be getting energy while performing no work. Using the coordinates shown in Fig. 1, where $r$ is the instantaneous length of the string and $R$ the radius of the finger, we have by the conservation of energy

$$r\dot{\theta}=\text{constant}.$$

Then the angular momentum is just proportional to $r$ and goes to zero as the string winds up. (The equations of motion are easily derived by using the relation $r=r_0-R\theta$.)

A proper illustration of the conservation of angular momentum is that of having the string exit through a hole in a smooth table. Here the radius may be shortened only by pulling on the end of the string. Angular momentum is conserved and the speed of the weight increases, but the resultant increase in kinetic energy is just equal to the work done by pulling the string. This correct illustration was used by Rossby[7] in a classic meteorological paper of two decades ago.

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[2] C. L. Stong, Sci. Am. **202**, 186 (June 1960).
[3] L. J. Battan, *The Nature of Violent Storms* (Doubleday and Company, New York, 1961), pp. 43, 44.
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[6] H. R. Byers, *General Meteorology* (McGraw-Hill Book Company, Inc., New York, 1959), p. 198.
[7] C. G. Rossby, "The Scientific Basis of Modern Meteorology," *U. S. Yearbook of Agriculture, "Climate and Man"* (U. S. Government Printing Office, Washington, D. C., 1941), p. 507.